THE ROTATION MATRIX FOR CALCULATING V2, V3 OFFSETS IN MODE 2 FOS TA

CAL/FOS-065

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March, 1990

INTRODUCTION

This is a check of the rotation matrices YFMTRXA and YFMTRXB used to calculate the V2, V3 offsets from the offsets calculated in FOS Mode 2 TA. Both the Binary Search and the Firmware use the same rotation matrix. The currently entered values of the matrixes are incorrect both in scale and in sign. (The only documentation on these matrixes is a letter from W. Bloomquist to Harms dated August 21 1980 which contains one error in angle between V2, V3 and FOS X, Y coordinates and has the sign reversed on the FOS Y coordinate.) The correct values are calculated.

I. CALCULATION OF ROTATION MATRIX

The rotation matrix transforms from FOS coordinates in units of $\left(\frac{1}{32}\right)\mu$ at the FOS detector photocathode, to V2, V3 coordinates in units of 2^{-27} radians. The matrix itself is scaled to 2^{-17} to accomodate the integer arithmetic performed by the NSSC-1.

$$\begin{pmatrix} V2 \\ V3 \end{pmatrix}_{2^{-27}rad} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_{2^{-17}} \begin{pmatrix} X \\ Y \end{pmatrix}_{(\frac{1}{32})\mu} \qquad (1)$$

The rotation matrix is simply the $cos\theta$, $sin\theta$ terms scaled to convert from $\left(\frac{1}{32}\right)\mu$ to 2^{-27} radians and multiplied by 2^{17}. In the rotation, the reversal of the telescope is taken into account.

$$SICS - TO - V2V3 = SCALE \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \qquad (2)$$

The conversion is given by the plate scale at the photocathode and the radian to arcseconds relation:

$$4.3 arcsec = 600\mu \tag{3}$$

$$\left(\frac{1}{32}\right)\mu = 2.23958 \times 10^{-4} arcsec \tag{4}$$

$$radians = 206,265 arcsec \tag{5}$$

$$1 = .145731 \left(\frac{2^{-27} radians}{\left(\frac{1}{32}\right)\mu}\right) \tag{6}$$

$$SCALE = 2^{17} \times 0.145731 \times \left(\frac{2^{-27} radian}{\left(\frac{1}{32}\right)\mu}\right) \tag{7}$$

$$SCALE = 1.90835 \times 10^{4} \left(\frac{2^{-27} radian}{\left(\frac{1}{32}\right)\mu}\right) \tag{8}$$

The scale is the same on the red and blue sides, but the angle θ is different. For the red side $\theta = -81.8102^\circ$ and for the blue side $\theta = -8.1898^\circ$

Thus the rotation matrix for the red side (without units written in the equation) is given by:

$$YFMTRXA = SCALE \begin{pmatrix} cos(-81.8102) & sin(-81.8102) \\ -sin(-81.8102) & cos(-81.8102) \end{pmatrix} \tag{9}$$

$$YFMTRXA = 1.90835 \times 10^{4} \begin{pmatrix} .14245 & -.98980 \\ .98980 & .14245 \end{pmatrix} \tag{10}$$

$$YFMTRXA = \begin{pmatrix} 2,718 & -18,889 \\ 18,889 & 2,718 \end{pmatrix}. \tag{11}$$

This can be compared to the present contents of the YFMTRXA matrix:

$$YFMTRXA = \begin{pmatrix} -2,658 & -18,915 \\ -18,915 & 2,658 \end{pmatrix} \tag{12}$$

The size of the numbers is about the same but the signs do not agree.

The correct matrix, in two's complement octal representation is:

$$YFMTRXA = \begin{pmatrix} 005236 & 733067 \\ 044711 & 005236 \end{pmatrix}. \tag{13}$$

For the blue side the rotation matrix is:

$$YFMTRXB = SCALE\begin{pmatrix} cos(-8.1898) & sin(-8.1898) \\ -sin(-8.1898) & cos(-8.1898) \end{pmatrix} \tag{14}$$

$$YFMTRXB = 1.90835 \times 10^4 \begin{pmatrix} .98980 & -.14245 \\ .14245 & .98980 \end{pmatrix} \tag{15}$$

$$YFMTRXB = \begin{pmatrix} 18,889 & -2,718 \\ 2,718 & 18,889 \end{pmatrix}. \tag{16}$$

This can be compared with the current contents of the YFMTRXB matrix:

$$YFMTRXB = \begin{pmatrix} -18,915 & -2,658 \\ -2,658 & 18,915 \end{pmatrix}. \tag{17}$$

The correct matrix in two's complement octal representation is:

$$YFMTRXB = \begin{pmatrix} 044711 & 772542 \\ 005236 & 044711 \end{pmatrix}. \tag{18}$$