FOS PROJECT DATA BASE APERTURE FILES

CAL/FOS-063

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ABSTRACT

The Project Data Base (PDB) science instrument aperture files (SIAF) contain all information necessary for placing a target of known position into the FOS apertures and for offsetting between different FOS apertures in a Mode I, real-time FOS target acquisition. We describe here the coordinate systems defined by the PDB and show where the coefficients for those coordinate systems are stored. We derive the contents of the PDB by starting from the blue side aperture positions measured at GSFC, and we summarize the sequence of measurements in OV and SV that will be used to update the PDB SIAF. A flow diagram is given in the Appendix for the path from measured parameters to PDB SIAF values.

I. INTRODUCTION

The aperture files are used to calculate offsets from targets found in the FOS acquisition aperture to science apertures. The telescope coordinate system has one axis looking out the telescope (V1), and two axes in the focal plane (V2, V3). The FOS apertures are about 4 arcminutes from the origin of the (V1, V2, V3). The PDB contains all parameters needed to transform from the FOS pixel location of the initial target acquisition image, to the V2, V3 telescope coordinate system. Note that the parameters in the aperture files are only for Mode I, real time FOS acquisitions. The Mode II FOS acquisitions are performed with different step sizes than Mode I, so the assumptions about pixel size used in the PDB would not be valid.

Figure 1 illustrates the relation between the V2-V3 coordinate system and a typical SICS or SIAS coordinate set. It is drawn with the V1 axis going into the paper so as to clearly illustrate the definition of the angle t. From this point of view, the rotation from V2 to V3 is clockwise, as is the X to Y rotation. Subsequent diagrams are drawn with V1 coming out of the paper and the V2-V3 and X-Y axes return to their conventional orientation. Changing the point of view does not change any of the transformation equations,
but causes the angle \( t \) to be shown as a clockwise rotation. Parity is positive when the rotation from \( X \) to \( Y \) is in the same direction as that from \( V2 \) to \( V3 \). Table 1a shows the parameters in the aperture file for the blue side target acquisition aperture and Table 1b shows the actual values of those parameters for the 4.3" target acquisition aperture. Table 2 summarizes the OV and SV tests relating to aperture location and plate scale.

**Homogeneous Coordinate Transformations**

For coordinate transformations in a plane, two by two matrices are adequate for describing rotations and scale changes, but do not permit a shift of the origin. The homogeneous coordinate approach allows origin shifts to be included in the transformation matrix. The concept is to replace the usual \( X, Y \) pair with a 3-dimensional vector \( X, Y, Z \) in which the physical \( X \) and \( Y \) positions are represented by \( X/Z \) and \( Y/Z \)

A transformation involving a shift of origin, a rotation and separate scale changes in each direction could be written as

\[
X' = a(X \cos \alpha + Y \sin \alpha) + X_0'
\]

(1)

\[
Y' = b(-X \sin \alpha + Y \cos \alpha) + Y_0'
\]

(2)

All of this can be incorporated in a homogeneous transformation of the form

\[
\begin{pmatrix}
X' \\
Y' \\
Z'
\end{pmatrix} =
\begin{pmatrix}
a \cos \alpha & a \sin \alpha & X_0' \\
-b \sin \alpha & b \cos \alpha & Y_0' \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
\]

(3)

where \( X_0', Y_0' \) is the origin of the \( X', Y' \) system in \( X, Y \) coordinates.

Taking the inverse of this matrix gives the correct inverse transformation and the third column shows the \( X, Y \) origin in terms of \( X', Y' \). The actual inverse matrix is:

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} =
\begin{pmatrix}
\cos \alpha/a & -\sin \alpha/b & Y_0' \sin \alpha/b - X_0' \cos \alpha/a \\
\sin \alpha/a & \cos \alpha/b & X_0' \sin \alpha/a - Y_0' \cos \alpha/b \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X' \\
Y' \\
Z'
\end{pmatrix}
\]

(4)
For these transformations, the value of $Z$ may be set equal to 1 and does not get changed. Parity reversal is implied if $a$ and $b$ are of opposite sign.

II. COORDINATE SYSTEMS DEFINED IN THE PDB

The contents of the PDB are defined in SE-06 section 10.4.6.1.1 and in ST ICD 26. The aperture files were first populated by O. Lupie. The aperture files in the PDB contain coefficients defining three coordinate systems. The first is simply the focal plane of the telescope as inverted by the telescope with the origin at the nominal center of the WF/PC CCDs. This system, with coordinates $V2, V3$, is in units of arcseconds. The FOS apertures, which are actually located in the $+V2, -V3$ quadrant, are referred to in the PDB aperture files as being in the $-V2, +V3$ quadrant because of the inversion of the telescope. The second coordinate system, Science Instrument Corrected System (SICS) has origins in the physical center of each aperture. SICS is also in units of arcseconds. Finally, the Science Instrument Aperture System (SIAS) is in units of FOS pixels (already restored, an issue that will be discussed later) and with an origin at the center of the 1,1 pixel. The SIAS system is based on the size of the FOS pixels, which depends on the Y-stepping and the X-stepping. As defined here, the SIAS system is relevant only for FOS images taken in ACQ Mode format.

The locations of the origins of these three coordinate systems are illustrated in Figure 2. A SICS system is defined for every aperture, with the origin in the physical center of the aperture. The SIAS coordinates defined for each image project onto the same place on the V2, V3 plane.

The transformations between the three systems, which will be referred to as $V2, V3$, $X_{SICS}, Y_{SICS}$, and $X_{SIAS}, Y_{SIAS}$, are given here as stated in SE-06-01 section 10.4.6.1.1 and ST ICD 26, chapter 7. First, we will discuss the transformation between $V2, V3$, $X_{SICS}, Y_{SICS}$. The SE-06-01 transformations imply that when parity reversal is applied, the sign of the angle tilt changes. Equations number 13, 14, 15, and 16 of SE-06-01 read (using our notation)

$$V2 = V2_0 + s \cos(tilt)X_{SICS} + s \sin(tilt)Y_{SICS} \quad (5)$$

$$V3 = V3_0 - \sin(tilt)X_{SICS} - \cos(tilt)Y_{SICS} \quad (6)$$

$$X_{SICS} = s \cos(tilt)(V2 - V2_0) - \sin(tilt)(V3 - V3_0) \quad (7)$$

$$Y_{SICS} = s \sin(tilt)(V2 - V2_0) + \cos(tilt)(V3 - V3_0) \quad (8)$$
where \( s \) is parity, \( \text{tilt} \) is the angle between \( V3 \) and the major axis (by definition \( Y \) axis) measured in a counter clockwise direction when looking along \( V1 \), and \( V2_0, V3_0 \) are the SICS origins in \( V2, V3 \).

Using homogeneous coordinate transformations, the SICS to \( V2, V3 \) transformation is

\[
\begin{pmatrix} V2 \\ V3 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\text{tilt}) & \sin(\text{tilt}) & V2_0 \\ -\sin(\text{tilt}) & \cos(\text{tilt}) & V3_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_{\text{SICS}} \\ Y_{\text{SICS}} \\ 1 \end{pmatrix}. \tag{9}
\]

Since there is no parity reversal between \( V2, V3 \) and FOS SICS, \( s \) has been included here as +1. There is no change of scale between \( V2, V3 \) and SICS since they are both in units of arcseconds.

![Figure 1: Orientation of V2V3 and SICS or SIAS axes. V1 axis goes into the page](image)

In the absence of distortion, the transformations between SICS and SIAS, in terms of the \( a, b, c, \) and \( d \) coefficients defined in SE-06-01, are given by:

\[
X_{\text{SICS}} = a_{1,1}X_{\text{SIAS}} + a_{1,0}Y_{\text{SIAS}} + a_{0,0} - a_{1,1}X_{a0} - a_{1,0}Y_{a0} \tag{10}
\]

\[
Y_{\text{SICS}} = b_{1,1}X_{\text{SIAS}} + b_{1,0}Y_{\text{SIAS}} + b_{0,0} - b_{1,1}X_{a0} - b_{1,0}Y_{a0} \tag{11}
\]

\[
X_{\text{SIAS}} = c_{1,1}X_{\text{SICS}} + c_{1,0}Y_{\text{SICS}} + c_{0,0} - c_{1,1}X_{c0} - c_{1,0}Y_{c0} \tag{12}
\]

\[
Y_{\text{SIAS}} = d_{1,1}X_{\text{SICS}} + d_{1,0}Y_{\text{SICS}} + d_{0,0} - d_{1,1}X_{c0} - d_{1,0}Y_{c0} \tag{13}
\]

The constants are each redundantly composed of three separate terms. If \((X_{a0}, Y_{a0})\) and \((X_{c0}, Y_{c0})\) are used, then \(a_{0,0}\) etc. are all zero. It would have been simpler to use just the \(a_{0,0}\) type of term which would have defined offsets in a more natural way.
Using homogeneous coordinate transformations, the SIAS to SICS transformation is

\[
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix} =
\begin{pmatrix}
a_{1,1} & a_{1,0} & a_{0,0} - a_{1,0} Y_{a0} - a_{1,1} X_{a0} \\
b_{1,1} & b_{1,0} & b_{0,0} - b_{1,0} Y_{a0} - b_{1,1} X_{a0} \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SIAS}} \\
Y_{\text{SIAS}} \\
1
\end{pmatrix}.
\tag{14}
\]

The inverse SICS to SIAS transformation is

\[
\begin{pmatrix}
X_{\text{SIAS}} \\
Y_{\text{SIAS}} \\
1
\end{pmatrix} =
\begin{pmatrix}
c_{1,1} & c_{1,0} & c_{0,0} - c_{1,0} Y_{c0} - c_{1,1} X_{c0} \\
d_{1,1} & d_{1,0} & d_{0,0} - d_{1,0} Y_{c0} - d_{1,1} X_{c0} \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix}.
\tag{15}
\]

The parameters given in the PDB aperture files are the values of \(\text{tilt}, V_2, V_3, a_{i,j}, b_{i,j}, c_{i,j},\) and \(d_{i,j}\). For the SIAS to SICS transformation, \(a_{1,0} = b_{1,1} = 0\) since the two systems are parallel, and \(a_{1,1} = b_{1,0} = \text{SCALE}\), where \(\text{SCALE}\) is nominally 0.0895"/pixel. Differences of several percent from the nominal are likely. Scale will be measured in SV. In the redundant definition of these parameters, \(a_{0,0} = b_{0,0} = c_{0,0} = d_{0,0} = 0\) always.

There is a complete set of these parameters for each aperture on both the red and the blue sides. There are nominally only 3 unique aperture positions on the red side and on the blue side while there are 15 different apertures on each side. The PDB values in this initial population of the SIAFs are therefore identical for many of the apertures. For example, the 4.3 \(\times\) 4.3 acquisition aperture has the same parameters as the 1.0" single, the 0.5" single, and the 0.3" single apertures while the upper paired apertures apertures all have the same parameters and the lower paired apertures all have the parameters. This can be seen in Figure 2, which shows the three unique aperture positions on the blue and the red side. Figure 3 illustrates the positions of the apertures (and SICS origins) in the SIAS coordinate system. Table 1 shows the placement of the coefficients in the PDB SIAF. Small differences in the actual location of apertures of the same type (e.g., single) may actually be present, and will be measured during OV.
# TABLE 1: PDB APERTURE FILES

**Parameters**

<table>
<thead>
<tr>
<th>V2(0)</th>
<th>V3(0)</th>
<th>RECT</th>
<th>4.3050nm 0.000TMMLCAJ 000350</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3050</td>
<td>SCALE</td>
<td>18.5360</td>
<td>TILT Y</td>
</tr>
<tr>
<td>11</td>
<td>X(a,0)</td>
<td>Y(a,0)</td>
<td>X(a,0)</td>
</tr>
<tr>
<td>a(0,0)</td>
<td>b(0,0)</td>
<td>c(0,0)</td>
<td>d(0,0)</td>
</tr>
<tr>
<td>a(1,0)</td>
<td>b(1,0)</td>
<td>c(1,0)</td>
<td>d(1,0)</td>
</tr>
<tr>
<td>a(1,1)</td>
<td>b(1,1)</td>
<td>c(1,0)</td>
<td>d(1,1)</td>
</tr>
<tr>
<td>V2(0)</td>
<td>V3(0)</td>
<td>0.0980</td>
<td>0.0980</td>
</tr>
<tr>
<td>90°TILT</td>
<td>TILT</td>
<td>X(a,0)</td>
<td>Y(a,0)</td>
</tr>
</tbody>
</table>

Values for the 4.3" Aperture

<table>
<thead>
<tr>
<th>V2 in &quot;</th>
<th>V3 in &quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>+33 5.1337</td>
<td>-397.45280</td>
</tr>
</tbody>
</table>

**Values for the 12.0" Aperture**

<table>
<thead>
<tr>
<th>V2(0)</th>
<th>V3(0)</th>
<th>RECT</th>
<th>4.3050nm 0.000TMMLCAJ 000350</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3050</td>
<td>164.5984</td>
<td>18.5360</td>
<td>-8.1898Y</td>
</tr>
<tr>
<td>11</td>
<td>23.5223</td>
<td>23.5223</td>
<td>-2.1053</td>
</tr>
<tr>
<td>FOS TBA-1 S</td>
<td>AJ 000380</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.00000</td>
<td>0.08950</td>
<td>0.00000</td>
<td>11.17318</td>
</tr>
<tr>
<td>0.08950</td>
<td>0.00000</td>
<td>11.17318</td>
<td>0.00000</td>
</tr>
<tr>
<td>-132.7700</td>
<td>164.5984</td>
<td>0.0895</td>
<td>0.0895</td>
</tr>
<tr>
<td>81.8011</td>
<td>-8.1898</td>
<td>23.52234</td>
<td>23.52234</td>
</tr>
</tbody>
</table>

1234567890123456789012345678901234567890123456789012345678901234567890
Figure 2. FOS Aperture Positions
Figure 3. SICS and SIAS Coordinates
III. DERIVATION OF PDB VALUES

The values used to derive all PDB entries the first time are: 1) $V_2$, $V_3$ positions of the center aperture on the blue side, 2) the angle between the FOS apertures and the $V_2$, $V_3$ coordinates (tilt), 3) the plate scale, 4) the pixel size, and 5) the distance between the upper and the lower apertures. In OV and SV, the aperture locations will be measured directly for all apertures and $tilt$ will be derived from those measurements.

Center Apertures

The $V_2$, $V_3$ positions for the center aperture used for this first derivation of the PDB were measured directly on the blue side using the quarterpanel at GSFC. The reference for this measurement is a Perkin Elmer memo dated September 10, 1985, from T.A. Facey to R.A. White at GSFC Code 400.2. The measured position of the blue aperture was $V_2 = 1.460$ inches and $V_3 = -1.810$ inches. The measured blue position is equal to the nominal position plus a small deviation, $\Delta V$.

$$V_{blue} = V_{nominal} + \Delta V$$

(16)

$$V_{blue} = \begin{pmatrix} 1.423 \\ -1.870 \end{pmatrix} + \begin{pmatrix} 0.037 \\ 0.060 \end{pmatrix}.$$  

(17)

The deviation from nominal position is assumed to be the same on the red side as on the blue side (i.e., the deviation is assumed to result from translation only, with no rotational error in the FOS orientation).

$$V_{red} = V_{nominal} + \Delta V$$

(18)

$$V_{red} = \begin{pmatrix} 1.870 \\ -1.423 \end{pmatrix} + \begin{pmatrix} 0.037 \\ 0.060 \end{pmatrix}.$$  

(19)

These positions, which are in units of inches, are converted to arcseconds by multiplying by 25.4mm/inch and by the plate scale 3.58025"/mm. The sign is reversed here because the PDB takes into account the inversion of the telescope.
\[
V_{\text{blue}} = \begin{pmatrix} -1.460 \\ 1.870 \end{pmatrix} \times 90.93835 = \begin{pmatrix} -122.7700 \\ 164.5984 \end{pmatrix}.
\tag{20}
\]

\[
V_{\text{red}} = \begin{pmatrix} -1.907 \\ 1.363 \end{pmatrix} \times 90.93835 = \begin{pmatrix} -173.4194 \\ 123.9490 \end{pmatrix}.
\tag{21}
\]

These \( V2, V3 \) values can be combined with the angle between the FOS apertures and the \( V2, V3 \) system (\textit{tilt}) to give the matrix elements of the transformation from SICS to \( V2, V3 \) (equation 5). The \textit{tilt} is \(-8.1898^\circ\) for the \textit{blue} side as measured from \( V3 \) to the \( Y \) axis where positive indicates counterclockwise viewed along the positive \( V1 \) axis (see FOS blueprint number 849AA132236 and the memo from G. Hartig to H. Ford dated March 11, 1985).

\[
\begin{pmatrix} V2 \\ V3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.9898 & -0.1425 & -132.7700 \\ 0.1425 & 0.9898 & 164.5984 \end{pmatrix} \begin{pmatrix} X_{\text{SICS}} \\ Y_{\text{SICS}} \\ 1 \end{pmatrix}.
\tag{22}
\]

The \textit{tilt} is \(-81.8102^\circ\) on the \textit{red} side, where the transformation matrix is

\[
\begin{pmatrix} V2 \\ V3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.1425 & -0.9898 & -173.4194 \\ 0.9898 & 0.1425 & 123.9490 \end{pmatrix} \begin{pmatrix} X_{\text{SICS}} \\ Y_{\text{SICS}} \\ 1 \end{pmatrix}.
\tag{23}
\]

Now the only thing remaining to completely populate the PDB for the central apertures is to define the relation between SICS and SIAS. SIAS is the aperture system in units of pixels, starting at pixel number 1,1. The FOS image is originally made up of (51,63) rectangular pixels, but after restoration in OSS, is made up of (48,48) square pixels, where the pixel size is a quarter of the width of an FOS diode, or 0.0895"/pixel. Figure 2 illustrates the relation between the SICS origin and the SIAS origin for the 4.3 \( \times \) 4.3 acquisition aperture. The distance from the SICS origin to the SIAS origin for both the blue and the red sides is given by

\[
X_{c,0} = \frac{-4.3''}{2} + \frac{0.358''}{8} = -2.10525''
\tag{24}
\]

\[
Y_{c,0} = \frac{-4.3''}{2} + \frac{0.358''}{8} = -2.10525''
\tag{25}
\]
This allows us to write down the transformations between SICS and SIAS.

\[
\begin{pmatrix}
X_{SICS} \\
Y_{SICS} \\
1
\end{pmatrix} =
\begin{pmatrix}
0.0895 & 0 & -2.10525 \\
0 & 0.0895 & -2.10525 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{SIAS} \\
Y_{SIAS} \\
1
\end{pmatrix}.
\] (26)

\[
\begin{pmatrix}
X_{SIAS} \\
Y_{SIAS} \\
1
\end{pmatrix} =
\begin{pmatrix}
11.1732 & 0 & 23.5223 \\
0 & 11.1732 & 23.5223 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{SICS} \\
Y_{SICS} \\
1
\end{pmatrix}.
\] (27)

**Upper and Lower Apertures**

The upper and lower apertures lie at the same angles as the center apertures, so their transformation matrices have the same values for the parameters \( \cos(tilt) \) and \( \sin(tilt) \), but their origins lie in different positions. The derivation of all the parameters for the upper and lower paired apertures involves only one additional piece of information; the distance between the paired apertures, which are located symmetrically above and below the center aperture. That distance is 3". The \( V_2, V_3 \) positions of the upper and lower apertures on both the blue and the red sides are given by

\[
V_{2_0}^{upper} = V_{2_0}^{center} + 1.5 \sin(tilt)
\] (28)

\[
V_{3_0}^{upper} = V_{3_0}^{center} - 1.5 \cos(tilt)
\] (29)

\[
V_{2_0}^{lower} = V_{2_0}^{center} - 1.5 \sin(tilt)
\] (30)

\[
V_{3_0}^{lower} = V_{3_0}^{center} - 1.5 \cos(tilt).
\] (31)

The SIAS origins for the upper apertures relative to the SICS origin for the center aperture for both the blue and the red sides are

\[
X_{c,0}^{upper} = X_{c,0}^{center}
\] (32)

\[
Y_{0,0}^{upper} = Y_{c,0}^{center} - 1.5.
\] (33)
The SIAS origins for the lower apertures relative to the SICS origin for the center aperture for both the blue and the red sides are

\[
X_{c,0}^{\text{lower}} = X_{c,0}^{\text{center}}
\]
\[
y_{c,0}^{\text{lower}} = y_{c,0}^{\text{center}} + 1.5.
\]

Equations 5, 6, and 7 can be used to completely describe the transformations from SIAS to SICS to V2, V3 coordinates. For the blue side upper apertures this is

\[
\begin{pmatrix}
V2 \\
V3 \\
1
\end{pmatrix} =
\begin{pmatrix}
0.9898 & -0.1425 & -132.9837 \\
0.1425 & 0.9898 & 166.0831 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix}.
\]
\[
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix} =
\begin{pmatrix}
0.0895 & 0 & -2.10525 \\
0 & 0.0895 & -3.60525 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SIAS}} \\
Y_{\text{SIAS}} \\
1
\end{pmatrix}.
\]

For the blue side lower apertures this is

\[
\begin{pmatrix}
V2 \\
V3 \\
1
\end{pmatrix} =
\begin{pmatrix}
0.9898 & -0.1425 & -132.5563 \\
0.1425 & 0.9898 & 163.1137 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix}.
\]
\[
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix} =
\begin{pmatrix}
0.0895 & 0 & -2.10525 \\
0 & 0.0895 & -0.60525 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SIAS}} \\
Y_{\text{SIAS}} \\
1
\end{pmatrix}.
\]

For the red side upper aperture this is

\[
\begin{pmatrix}
V2 \\
V3 \\
1
\end{pmatrix} =
\begin{pmatrix}
0.1425 & -0.9898 & -174.9041 \\
0.9898 & 0.1425 & 124.1627 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix}.
\]
\[
\begin{pmatrix}
X_{\text{SICS}} \\
Y_{\text{SICS}} \\
1
\end{pmatrix} =
\begin{pmatrix}
0.0895 & 0 & -2.10525 \\
0 & 0.0895 & -3.60525 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X_{\text{SIAS}} \\
Y_{\text{SIAS}} \\
1
\end{pmatrix}.
\]
For the red side lower apertures this is

$$\begin{pmatrix}
V2 \\
V3 \\
1
\end{pmatrix} = \begin{pmatrix}
0.1425 & -0.9898 & -171.9347 \\
0.9898 & 0.1425 & 123.7353 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
X_{SICS} \\
Y_{SICS} \\
1
\end{pmatrix}. \quad (42)$$

$$\begin{pmatrix}
X_{SICS} \\
Y_{SICS} \\
1
\end{pmatrix} = \begin{pmatrix}
0.0895 & 0 & -2.10525 \\
0 & 0.0895 & -0.60525 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
X_{SIAS} \\
Y_{SIAS} \\
1
\end{pmatrix}. \quad (43)$$

IV. UPDATING PDB SIAF FROM OV AND SV TESTS

Aperture positions are measured in OV first crudely for the large aperture, and then more accurately for the large aperture, and finally finely for the small apertures. Relative aperture positions and areas are measured and used together with the small aperture positions to give positions of all apertures. Thus, the aperture positions for the upper and lower paired apertures are measured directly in OV rather than being derived from the tilt plus the aperture separation as in Section III.

The first measurement of $V2, V3$ position of the large 4.3" aperture is performed in OV 1442 in 3" steps, and the result will be used to update the PDB for OV 1443. Then, the 4.3" position is measured in 0.3" steps in OV 1443. The results from OV 1443 will be used to update the PDB for OV 1444, which measures the $V2, V3$ positions for the upper and lower 0.1" apertures in 0.02" steps and for the .25" lower aperture in 0.04" steps. OV 1444 will also provide the most precise locations for the larger apertures when linked with the results of OV 1441, which measures relative positions of all FOS apertures. Table 2 summarizes the results obtained from OV and also from SV tests. The OV and SV tests are summarized below.

**OV 1441 Aperture Locations Phase I. Relative Positions**

OV 1441 measures relative aperture location in detector coordinates. The relative positions will be combined with the accurate positions measured in OV 1444 for the 0.1 upper and lower apertures to give positions for most of the FOS apertures. OV 1441 is the only measurement of the positions and areas of most FOS apertures.
OV 1442 Aperture Location Phase II. 4.3 Aperture Position

OV 1442 is the first crude attempt at placing a star in the 4.3" target acquisition aperture. The test places a star on a 12 × 12 grid in 3" steps and will result in a coarse V2, V3 position, and coarse values for plate scale and tilt. The results will be used to update the PDB for OV 1443.

OV 1443 Aperture Location Phase IIIa. 4.3 Position

OV 1443 maps the 4.3 aperture at a series of different pointings perpendicular to the edges. The star is stepped over a length of 0.5" with step sizes of 0.03", once over each edge. The resulting V2, V3 position for the 4.3" aperture will be used to update the PDB for OV 1444.

OV 1444 Aperture Location Phase IIIb. Fine Positions

OV 1444 measures the locations of the 0.1" upper and lower apertures and the 0.25" lower aperture by rastering a star on a fine grid to find the maximum throughput. The grid for the 0.1" apertures is 0.1 × 0.1 with 0.02" steps, while the grid for the 0.25" lower aperture is 0.24 × 0.24 with 0.04" steps. OV 1444 provides the best measurement of V2, V3 for the 0.1" apertures as well as a direct measurement of plate scale and tilt. The results of OV 1444 will be combined with the results of OV 1441 to give the most precise measurement of the locations of the larger FOS apertures.

OV 1527 Aperture Location Phase IVa. Fine Positions

SV 1527 is the same as OV 1444. SV 1527 is repeated 5 times in SV. Any changes in V2, V3 positions of the 0.1" apertures in SV will result in updates to the PDB SIAF.

OV 1528 Aperture Location Phase IVb. Fine Plate Scale

A step and dwell sequence is performed and the 4.3" aperture is mapped at each position in the sequence. The pattern is a 4 × 4 grid with 1" steps. SV 1528 measures plate scale and tilt.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>OV Crude</th>
<th>OV Fine</th>
<th>SV</th>
</tr>
</thead>
<tbody>
<tr>
<td>plate scale</td>
<td>1442</td>
<td>1444</td>
<td>1527, 1528</td>
</tr>
<tr>
<td>tilt</td>
<td>1442</td>
<td>1444</td>
<td>1527, 1528</td>
</tr>
<tr>
<td>4.3</td>
<td>1442</td>
<td>1443</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>1.0-PAIR A,B</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>0.5-PAIR A,B</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>0.25-PAIR A</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>0.25-PAIR B</td>
<td></td>
<td>1444</td>
<td>1527</td>
</tr>
<tr>
<td>0.1-PAIR A,B</td>
<td></td>
<td>1444</td>
<td>1527</td>
</tr>
<tr>
<td>0.7X2.0-BAR</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>2.0-BAR</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
<tr>
<td>0.25X2.0</td>
<td></td>
<td>1444+1441</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX

The appendix shows worksheets generated by the program MATHCAD. The manner in which PDB quantities are generated from original measurements is made clear. Near the bottom of each page is an array of numbers which model the layout of the entries in the SIAF files of the PDB.
ORIGIN := 1  Array indices start at 1

INPUT VALUES

scale := 0.0895  arcsec per pixel
tilt := -8.1898  degrees
Aperture YBL4 3  NECT  4.3 by 4.3
V2C := -132.7700  V3C := 164.5984
XC00 := -2.10525  YC00 := -2.10525  SSIAS origin in SICS coordinates

DERIVED VALUES

V2 := V2C  V3 := V3C

\[
\begin{bmatrix}
\cos(\text{tilt}) & \sin(\text{tilt}) & V2 \\
-\sin(\text{tilt}) & \cos(\text{tilt}) & V3 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sics\_to\_v2v3 := \\
0.99  -0.142  -132.77 \\
0.142  0.99  164.598 \\
0  0  1
\]

\[
\begin{bmatrix}
\text{scale} & 0 & XC0 \\
0 & \text{scale} & YC0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sias\_to\_sics := \begin{bmatrix}
0.09 & 0 & -2.105 \\
0 & 0.09 & -2.105 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sics\_to\_sias := sias\_to\_sics
\]

\[
sics\_to\_sias := \begin{bmatrix}
11.173 & 0 & 23.522 \\
0 & 11.173 & 23.522 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
P := sics\_to\_sias  \quad Q := sias\_to\_sics  \quad R := sics\_to\_v2v3
\]

\[
\begin{bmatrix}
P  & P \\
1,3 & 2,3 & XC0 & YC0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
1,2 & 2,2 & 1,2 & 2,2 \\
1,1 & 2,1 & 1,1 & 2,1 \\
1,3 & 2,3 & 1,1 & 1,1 \\
90 + \text{tilt} & \text{tilt} & 1,3 & 2,3
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0.09 & 0 & 11.173 \\
-132.77 & 164.598 & 0.09 & 0.09 \\
81.81 & -8.19 & 23.522 & 23.522
\end{bmatrix}
\]

\[
\text{CAJ3} := (23.522 23.522 -2.105 -2.105)
\]

\[
PDB =
\]

\[
\text{CAJ3} =
\]
Aperture YBL0 1PRA  RECT  0.1 by 0.1
Position of upper aperture calculated from position of center aperture
1.5 arcsec from center along axis at angle
tilt = 8.19 degrees

V2 := V2C + 1.5 \cdot \sin(\text{rtilt}) \quad V3 := V3C + 1.5 \cdot \cos(\text{rtilt})

\[
\begin{bmatrix}
\cos(\text{rtilt}) & \sin(\text{rtilt}) & V2 \\
-\sin(\text{rtilt}) & \cos(\text{rtilt}) & V3 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sics\_to\_V2V3 = \begin{bmatrix}
0.99 & -0.142 & -132.984 \\
0.142 & 0.99 & 166.083 \\
0 & 0 & 1
\end{bmatrix}
\]

YCO := YC00 - 1.5

\[
sias\_to\_sics := \begin{bmatrix}
scale & 0 & XCO \\
0 & scale & YCO \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sias\_to\_sics = \begin{bmatrix}
0.09 & 0 & -2.105 \\
0 & 0.09 & -3.605 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sics\_to\_sias := sias\_to\_sics
\]

\[
sics\_to\_sias = \begin{bmatrix}
11.173 & 0 & 23.522 \\
0 & 11.173 & 40.282 \\
0 & 0 & 1
\end{bmatrix}
\]

P := sics\_to\_sias \quad Q := sias\_to\_sics \quad R := sics\_to\_V2V3

\[
\begin{bmatrix}
P & P \\
1.3 & 2.3 & XCO & XCO
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
Q & Q & P & P \\
1,2 & 2,2 & 1,2 & 2,2 \\
Q & Q & P & P \\
1,1 & 2,1 & 1,1 & 2,1 \\
R & R & Q & Q \\
1,3 & 2,3 & 1,1 & 1,1 \\
P & P
\end{bmatrix}
\]

\[
PDB := \begin{bmatrix}
90 + \text{tilt} & \text{tilt} & 1,3 & 2,3
\end{bmatrix}
\]

\[
CAJ3 = \begin{bmatrix}
(23.522 & 40.282 & -2.105 & -3.605) \\
0 & 0 & 0 & 0 \\
0 & 0.09 & 0 & 11.173 \\
0.09 & 0 & 11.173 & 0 \\
-132.984 & 166.083 & 0.09 & 0.09 \\
81.81 & -8.19 & 23.522 & 40.282
\end{bmatrix}
\]
Lower aperture YBL0_1PRB  RECT  0.1 by 0.1

\[ V2 := V2C - 1.5 \cdot \sin(\text{rtilt}) \quad V3 := V3C - 1.5 \cdot \cos(\text{rtilt}) \]

\[
sics_{\text{to V2V3}} := \begin{bmatrix}
\cos(\text{rtilt}) & \sin(\text{rtilt}) & V2 \\
-sin(\text{rtilt}) & \cos(\text{rtilt}) & V3 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sics_{\text{to V2V3}} = \begin{bmatrix}
0.99 & -0.142 & -132.556 \\
0.142 & 0.99 & 163.114 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ YCO := YCO0 + 1.5 \]

\[
sias_{\text{to sics}} := \begin{bmatrix}
scale & 0 & YCO \\
0 & scale & YCO \\
0 & 0 & 1
\end{bmatrix}
\]

\[ sias_{\text{to sics}} = \begin{bmatrix}
0.09 & 0 & -2.105 \\
0 & 0.09 & -0.605 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ sics_{\text{to sias}} := sias_{\text{to sics}}^{-1} \]

\[ sics_{\text{to sias}} = \begin{bmatrix}
11.173 & 0 & 23.522 \\
0 & 11.173 & 6.763 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ P := sics_{\text{to sias}} \quad Q := sias_{\text{to sics}} \quad R := sics_{\text{to V2V3}} \]

\[ CAJ3 := \begin{bmatrix}
1,3 & 2,3 & XCO & YCO
\end{bmatrix}
\]

\[ CAJ3 = \begin{bmatrix}
11.173 & 0 & 23.522 \\
0 & 0.09 & 11.173 \\
0.09 & 0 & 11.173 \\
-132.556 & 163.114 & 0.09 & 0.09 \\
81.81 & -8.19 & 23.522 & 6.763
\end{bmatrix}
\]

\[ PDB := \begin{bmatrix}
0 & 0 & 0 & 0 \\
1,2 & 2,2 & 1,2 & 2,2 \\
1,1 & 2,1 & 1,1 & 2,1 \\
1,3 & 2,3 & 1,1 & 1,1
\end{bmatrix}
\]

\[ PDB = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0.09 & 0 & 11.173 \\
0.09 & 0 & 11.173 & 0 \\
-132.556 & 163.114 & 0.09 & 0.09 \\
81.81 & -8.19 & 23.522 & 6.763
\end{bmatrix}
\]
FOS APERTURES RED SIDE

ORIGIN := 1 Array indices start at 1

INPUT VALUES
scale := 0.0895 arcsec per pixel
tilt := -81.8102 degrees
Aperture YRD4 3 RECT 4.3 by 4.3
V2C := -173.4194 V3C := 123.9490
XC00 := -2.10525 YC00 := -2.10525 SIAS origin in SICS coordinates

DERIVED VALUES
V2 := V2C V3 := V3C

\[
sics_to_V2V3 := \begin{bmatrix}
\cos(\text{rtilt}) & \sin(\text{rtilt}) & V2 \\
-\sin(\text{rtilt}) & \cos(\text{rtilt}) & V3 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\text{rtilt} := \frac{\pi}{180} \times \text{tilt radians}
\]

\[
sics_to_V2V3 = \begin{bmatrix}
0.142 & -0.99 & -173.419 \\
0.99 & 0.142 & 123.949 \\
0 & 0 & 1
\end{bmatrix}
\]

XC0 := XC00 YC0 := YC00

\[
sias_to_sics := \begin{bmatrix}
\text{scale} & 0 & \text{XC0} \\
0 & \text{scale} & \text{YC0} \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sias_to_sics = \begin{bmatrix}
0.09 & 0 & -2.105 \\
0 & 0.09 & -2.105 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
sics_to_sias := \text{sias_to_sics}^{-1}
\]

\[
sics_to_sias = \begin{bmatrix}
11.173 & 0 & 23.522 \\
0 & 11.173 & 23.522 \\
0 & 0 & 1
\end{bmatrix}
\]

P := sics_to_sias Q := sics_to_sics R := sics_to_V2V3

\[
\text{CAJ3} := \begin{bmatrix}
P & P \\
1,3 & 2,3 & \text{XC0} & \text{YC0}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1,2 & 2,2 & 1,2 & 2,2 & P \\
Q & Q & P & P \\
1,1 & 2,1 & 1,1 & 2,1 & R \\
Q & Q & P & P \\
1,3 & 2,3 & 1,1 & 1,1 & \text{P} & \text{P} \\
90 + \text{tilt} & \text{tilt} & 1,3 & 2,3 & \text{P} & \text{P}
\end{bmatrix}
\]

\[
\text{CAJ3} = \begin{bmatrix}
23.522 & 23.522 & -2.105 & -2.105 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0.09 & 0 & 11.173 & 0 \\
-173.419 & 123.949 & 0.09 & 0.09 & 0 \\
8.19 & -81.81 & 23.522 & 23.522 & 0
\end{bmatrix}
\]

\[
\text{PDB} := \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0.09 & 0 & 11.173 & 0 \\
-173.419 & 123.949 & 0.09 & 0.09 & 0 \\
8.19 & -81.81 & 23.522 & 23.522 & 0
\end{bmatrix}
\]
Aperture YRDO_1PR4 RECT 0.1 by 0.1
Position of upper aperture calculated from position of center aperture
1.5 arcsec from center along axis at angle
tilt = -81.81 degrees

\[ V2 := V2C + 1.5 \cdot \sin(r_{tilt}) \quad V3 := V3C + 1.5 \cdot \cos(r_{tilt}) \]

\[
sics\_to\_V2V3 := \begin{bmatrix} \cos(r_{tilt}) & \sin(r_{tilt}) & V2 \\ -\sin(r_{tilt}) & \cos(r_{tilt}) & V3 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
sics\_to\_V2V3 = \begin{bmatrix} 0.142 & -0.99 & -174.904 \\ 0.99 & 0.142 & 124.163 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[ XC0 := XC00 - 1.5 \]

\[
sias\_to\_sics := \begin{bmatrix} \text{scale} & 0 & XC0 \\ 0 & \text{scale} & YC0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{sias\_to\_sics} = \begin{bmatrix} 0.09 & 0 & -3.605 \\ 0 & 0.09 & -2.105 \\ -1 & 0 & 0 \end{bmatrix}
\]

\[
sics\_to\_sias := sias\_to\_sics \quad \text{sics\_to\_sias} = \begin{bmatrix} 0 & 0 & 11.173 & 0 & 40.282 \\ 0 & 11.173 & 23.522 \end{bmatrix}
\]

\[ P := \text{sics\_to\_sias} \quad Q := \text{sias\_to\_sics} \quad R := \text{sics\_to\_V2V3} \]

\[ \text{CAJ3} := \begin{bmatrix} P & P \\ 1,3 & 2,3 & XC0 & YC0 \end{bmatrix} \]

\[ \text{PDB} := \begin{bmatrix} 0 & 0 & 0 & 0 \\ Q & Q & P & P \\ 1,2 & 2,2 & 1,2 & 2,2 \\ Q & Q & P & P \\ 1,1 & 2,1 & 1,1 & 2,1 \\ R & R & Q & Q \\ 1,3 & 2,3 & 1,1 & 1,1 \\ 90 + \text{tilt} & \text{tilt} & 1,3 & 2,3 \end{bmatrix} \]

\[ \text{CAJ3} = (40.282 \ 23.522 \ -3.605 \ -2.105) \]

\[ \text{FDB} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 11.173 \\ 0.09 & 0.09 & 0 & 11.173 \\ -174.904 & 124.163 & 0.09 & 0.09 \\ 8.19 & -81.81 & 40.282 & 23.522 \end{bmatrix} \]
Lower aperture YRD0_1PRB  RECT  0.1 by 0.1

\[ V2 := V2C - 1.5 \cdot \sin(\text{tilt}) \quad V3 := V3C - 1.5 \cdot \cos(\text{tilt}) \]

\[
sics\_to\_V2V3 := \begin{bmatrix} \cos(\text{tilt}) & \sin(\text{tilt}) & V2 \\ -\sin(\text{tilt}) & \cos(\text{tilt}) & V3 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
sics\_to\_V2V3 = \begin{bmatrix} 0.142 & -0.99 & -171.935 \\ 0.99 & 0.142 & 123.735 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[ XCO := XCO0 + 1.5 \]

\[
sias\_to\_sics := \begin{bmatrix} \text{scale} & 0 & XCO \\ 0 & \text{scale} & YCO \\ 0 & 0 & 1 \end{bmatrix} \quad \text{sias\_to\_sics} = \begin{bmatrix} 0.09 & 0 & -0.605 \\ 0 & 0.09 & -2.105 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
sics\_to\_sias := sias\_to\_sics
\]

\[
sics\_to\_sias = \begin{bmatrix} 11.173 & 0 & 6.763 \\ 0 & 11.173 & 23.522 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[ P := sics\_to\_sias \quad Q := sias\_to\_sics \quad R := sics\_to\_V2V3 \]

\[ \text{CAJ3} := \begin{bmatrix} P & P \\ 1,3 & 2,3 & XCO & YCO \end{bmatrix} \]

\[
\text{PDB} := \begin{bmatrix} 0 & 0 & 0 & 0 \\ Q & Q & P & P \\ 1,2 & 2,2 & 1,2 & 2,2 \\ Q & Q & P & P \\ 1,1 & 2,1 & 1,1 & 2,1 \\ R & R & Q & Q \\ 1,3 & 2,3 & 1,1 & 1,1 \\ \text{tilt} & \text{tilt} & 1,3 & 2,3 \end{bmatrix}
\]

\[ \text{CAJ3} = (6.763 \ 23.522 \ -0.605 \ -2.105) \]

\[
\text{PDB} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 11.173 \\ 0.09 & 0 & 11.173 & 0 \\ -171.935 & 123.735 & 0.09 & 0.09 \\ 8.19 & -81.033 & 6.763 & 23.522 \end{bmatrix}
\]